Signatures of Strong Correlations in One-Dimensional Ultra-Cold Atomic Fermi Gases

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Recent success in manipulating ultra-cold atomic systems allows to probe different strongly correlated regimes in one-dimension. Regimes such as the (spin-coherent) Luttinger liquid and the spin-incoherent Luttinger liquid can be realized by tuning the inter-atomic interaction strength and trap parameters. We identify the noise correlations of density fluctuations as a robust observable (uniquely suitable in the context of trapped atomic gases) to discriminate between these two regimes. Finally, we address the prospects to realize and probe these phenomena experimentally using optical lattices.

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Strong correlations in low-dimensional Fermi systems give rise to interesting many-body states such as those responsible for High-T_c superconductivity and fractional quantum Hall effect (FQHE) in two dimensions (2D), and Luttinger liquid [1, 2] in one dimension (1D). In particular, the Luttinger liquid phase has been the paradigm of low-energy physics in 1D systems for about half a century [3, 4]. This phase is characterized by the absence of fermionic quasi-particles even in the presence of a well defined Fermi surfaces with the relevant modes of dispersion represented by bosonic spin and charge excitations propagating at different velocities (spin-charge separation). Very recently, a new regime, the spin-incoherent (SI) Luttinger liquid [5, 6], has become an active area of research. In contrast with the spin-coherent (SC) Luttinger liquid, here the spin-incoherence results from the induced spin-spin interactions scale $J = 2\pi^2\hbar^4 \langle\rho\rangle^3 / (3m^2 U_{1\mathrm{D}})$ [7] being the smallest scale in the system, $J \ll T \ll E_F$, where E_F is the Fermi energy, T the temperature, $\langle\rho\rangle$ the total density and $U_{1\mathrm{D}}$ the interaction strength for contact interactions. In systems of ultra-cold atoms, this regime can be reached by increasing either the interaction strength among particles or by reducing the density. Thus, even at extremely low temperatures, fluctuations may drive the Luttinger liquid to lose spin coherence and only charge excitations remain as the dominant propagating mode. While there exists a growing experimental evidence in support for the SC Luttinger liquid [2], experimental evidence for the SI Luttinger liquid is very scarce [8, 9]. Even though it is possible to change density in 1D condensed matter systems by applying a gate voltage, lack of tunability in the interaction strength limits its possible experimental realizations. In contrast, in trapped ultra-cold atomic systems the SI Luttinger liquid regime is unavoidable since low-densities are inevitably reached near the confinement edges.

With the recent developments in the techniques of trapping and manipulation of ultra-cold atomic gases, the study of the strongly correlated regime of many-body systems has acquired new momentum. Trapped atoms form an ideal many-body system that can be configured with extreme control and purity allowing for a thorough study of many-body properties previously inaccessible in solid state configurations. Experimentalists are able to achieve strongly correlated regimes either by configuring the interaction to be strong by use of a tunable atom-atom scattering resonance, for example a magnetic Feshbach resonance [10], or by introducing degeneracy in the single particle ground state by creating a lattice trapping potential [11]. The former occurs when a two-body bound molecular states is made resonant with the open channel threshold by dialing-in an external magnetic field. This allows for the effective two-body interaction to have a negative (attractive) or positive (repulsive) sign with a strength that is dependent on the distance of the bound state from the scattering threshold. The lattice potential is induced by creating a standing wave pattern when two laser beams interfere at an angle [11]. The depth of the individual lattice well and the distance between wells is directly related to the intensity and the wavelength of the laser light. Such controllability made available by optical lattices has played an indispensable role in the trapped-dilute-gas demonstration of condensed matter phenomena such as superfluid to Mott insulator transition [12]. In other experiments, multiple beams were used to create an optical lattice in two spatial dimensions, for example x and y, resulting in a configuration consisting of an x-y array of tubes as depicted in Fig. 1. Each tube can act as an independent elongated quasi-1D trap if the

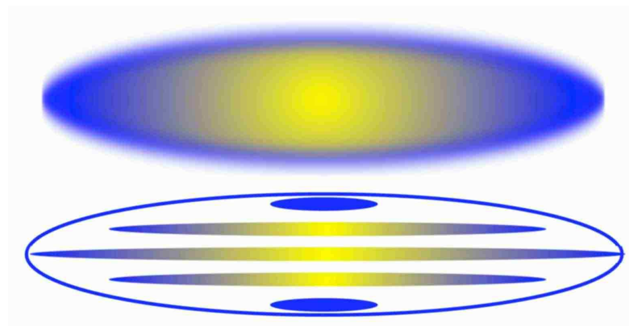

FIG. 1: Schematic sketch of the 3D cigar-shaped cloud (upper) and array of 1D tubes (lower) formed by optical trap lattice potentials. Yellow (light) regions represent the spin-coherent Luttinger liquid regime, while blue (dark) regions indicate its spin-incoherent counterpart.

tunneling between the tubes is adjusted to be zero with the transverse trapping frequency large compared to all scales, including interaction energy scale. Such controlled 1D-trap arrays allowed for the successful demonstration of the Tonks-Girardeau Gas regime [13, 14] and play a key role in theoretical proposals of achieving a dilute-gas SC Luttinger liquid [15, 16]. Moreover, trapped multi-species/spin atomic systems offer the added advantage of individual addressability. This allows for measurement of properties that are species/spin dependent. For example, it is possible to measure the spin up-up and down-down correlation function separately.

In this Letter, we extend the 1D Fermi gas studies to both regimes of the Luttinger liquid in two-component quasi-1D Fermi gas as confined in an array of tubes as shown in Fig. 1. Previous theoretical studies of this regime in ultra-cold Fermi gases have been limited to investigating the separation between charge and spin degrees of freedom [15, 16]. Here, we focus on density fluctuations and show that there is a qualitative difference between the SC Luttinger liquid and its spin-incoherent counterpart.

We consider a two-component ultra-cold atomic gas in a 3D magnetic trap. To define quasi-1D tubes, an optical lattice is imposed in the transverse direction. The combined effect of the trap potential can be modeled by $V(\vec{x}) = m\omega_\perp^2(x^2+y^2)/2 + m\omega_z^2 z^2/2 + V_0\left[\sin^2(2\pi x/\lambda) + \sin^2(2\pi y/\lambda)\right]$. Here, $\omega_\perp, \omega_z$ are the trap frequencies for the cigar-shaped ($\omega_z < \omega_\perp$) trap, $\lambda/2$ is the period of the optical lattice and V_0 is its depth. The interaction between atoms is modeled by the contact pseudopotential $V_{\rm int}(\vec{x}_1 - \vec{x}_2) = U_{\rm 3D}\delta(\vec{x}_1 - \vec{x}_2)$, with $U_{\rm 3D} = 4\pi\hbar^2 a_{\rm 3D}/m$, where $a_{\rm 3D}$ is the 3D s-wave scattering length. For large depth of the optical potential, the 3D cloud splits up into 1D tubes and each tube is then described by the following 1D Hamiltonian:

$$
\begin{aligned}
H_{\rm 1D} \;&=\; \sum_{\sigma=\uparrow,\downarrow}\int dz\psi_\sigma^\dagger(z)\left[\frac{p_z^2}{2m} + V_{\rm trap}(z)\right]\psi_\sigma(z) + \\
&+\; U_{\rm 1D}\int dz\psi_\uparrow^\dagger(z)\psi_\downarrow^\dagger(z)\psi_\downarrow(z)\psi_\uparrow(z), \qquad (1)
\end{aligned}
$$

where $\psi_\sigma^\dagger(\vec{x})$ [$\psi_\sigma(\vec{x})$] is the creation [annihilation] operator for particles of spin σ. $V_{\rm trap}(z) = m\omega_z^2 z^2/2$ is the 1D trap potential and $U_{\rm 1D} = -2\hbar^2/ma_{\rm 1D}$ represents the effective 1D interaction strength defined in terms of the effective 1D scattering length $a_{\rm 1D} = -(a_\perp^2/a_{\rm 3D})(1 - Ca_{\rm 3D}/a_\perp)$, with $C = |\zeta(1/2)|/\sqrt{2}$ [17, 18]. As we see, the 1D interaction strength exhibits a "geometric" resonance for a certain 3D scattering length, that can be employed to tune it by varying magnetic field. Here $a_\perp = (\hbar/m\omega_0)^{1/2}$ is the harmonic oscillator length in the transverse direction for the quasi-1D tubes with $\omega_0 = (8\pi^2 V_0/m\lambda^2)^{1/2}$. The number of particles in each tube is determined by the global trap imposed on the 3D cloud. To justify the above continuum description one should demand that all relevant energy scales of the system (collectively denoted by E), which can be tuned to a great extent, obey the inequality $\hbar\omega_z \ll E \ll \hbar\omega_0$.

Now, we begin by discussing the homogeneous situation *i.e.*, $V_{\rm trap} = 0$. The model given by Eq. (1) (and its lattice variant, known as the Hubbard model) has been exactly solved by Bethe Ansatz [19] and it is known that for repulsive interactions the low-energy and long-wavelength physics is described by the SC Luttinger liquid, with two copies of non-interacting charge and spin bosons, and is governed by the Hamiltonian [1]:

$$
H_{\rm SC} = \sum_{\alpha=c,s}\frac{v_\alpha}{2}\int dz\left[K_\alpha\Pi_\alpha^2 + \frac{1}{K_\alpha}(\partial_z\varphi_\alpha)^2\right], \qquad (2)
$$

which describes the fluctuations of the charge and spin densities above the ground state values. The bosonic field operators obey canonical commutation relations $[\varphi_\alpha(z), \Pi_{\alpha'}(z')] = i\delta_{\alpha\alpha'}\delta(z-z')$ and $[\varphi_\alpha(z), \varphi_{\alpha'}(z')] = [\Pi_\alpha(z), \Pi_{\alpha'}(z')] = 0$, where $\Pi_\alpha = \frac{1}{K_\alpha v_\alpha}\partial_t\varphi_\alpha$ is the momentum conjugate to the bosonic field φ_α and $v_{c(s)}$ is charge (spin) velocity. For spin-rotation invariant systems, the SC Luttinger liquid interaction parameter in the spin sector is $K_s = 1$ [1]. The spin and charge velocities and the interaction parameter in the charge sector can be determined from the exact solution for any interaction strength and density [20]. In the weak coupling regime, they can also be obtained by bosonization [1]. In contrast, the SI Luttinger liquid is described only by the charge bosonic field [21, 22]. The Hamiltonian in this regime, $H_{\rm SI}$, can be easily obtained from Eq. (2) by dropping the spin part and making the substitutions: $K_c = 1/2$ and $v_c = 2v_F$ (which is exact for local interactions). We assumed that the temperature is very small compared to Fermi energy and considered the limit $J \to 0$ first and then $T \to 0$ (opposite order of limits than in the SC Luttinger liquid). The behavior is equivalent to that of non-interacting spinless fermions in agreement with the exact results [19].

We shall now include the effects of the trap on a quasi-1D tube via local density (Thomas-Fermi) approximation. Thus, the average density of the system as a function of coordinate can be found as a solution of the equation

$$
\frac{dE[\langle\rho\rangle]}{d\langle\rho\rangle} = \mu - V_{\rm trap}(z), \qquad (3)
$$

where $E[\langle\rho\rangle]$ is the ground-state energy of the uniform system per unit length and μ is the chemical potential fixed by normalization, $\int\langle\rho(z)\rangle\, dz = N$. In the SC Luttinger liquid regime, for weak interactions (near the center of the trap) [15]

$$
E_{\rm SC}[\langle\rho\rangle] = \frac{\hbar^2\pi^2\langle\rho\rangle^3}{24m} + \frac{U_{1D}\langle\rho\rangle^2}{4}, \qquad (4)
$$

then from Eq. (3) the density reads

$$\langle\rho\rangle_{\mathrm{SC}}(z) = \langle\rho_0\rangle \left(\sqrt{1+g^2-\frac{z^2}{R^2}}-g\right), \quad |z| \le R, \quad (5)$$

where $R = (2\mu/m\omega_z^2)^{1/2}$, $\langle\rho_0\rangle = (8m\mu/\hbar^2\pi^2)^{1/2}$ is the density of the non-interacting uniform system and $g = U_{1D}/\hbar\pi v_{F0}$, with $v_{F0} = \hbar\pi\langle\rho_0\rangle/2m$. Due to the coordinate dependence of the density of particles, the spin and charge velocities, as well as the Luttinger liquid interaction parameters become coordinate dependent [15]. From bosonization results we can write that $v_s(z) = v_F(z) - U_{1D}/2\pi\hbar$, $v_c(z) = v_F(z)/K_c(z)$ and $K_c(z) = (1 + U_{1D}/\hbar\pi v_F(z))^{-1/2}$. These results are valid for weak coupling only, which is apparent since the expressions break down near the trap edges where the interactions become stronger. Since the trap potential does not break spin rotation invariance, $K_s(z) = 1$. For strong interactions, the SC Luttinger liquid regime can only be realized near the trap center. Thus, we will approximate $K_c(z)$ to its value at $z = 0$, $K_c(0) = K_{c,0}$; cf. Ref. [23].

We can do similar calculations for the spin-incoherent regime with energy [15]

$$E_{\mathrm{SI}}[\langle\rho\rangle] = \frac{\hbar^2\pi^2\langle\rho\rangle^3}{6m}, \quad (6)$$

since in this limit atoms behave as spinless particles. Using Eq. (3) again, the density reads this time

$$\langle\rho\rangle_{\mathrm{SI}}(z) = \langle\rho_0\rangle\sqrt{1-\frac{z^2}{R^2}}, \qquad |z| \le R, \quad (7)$$

where $\langle\rho_0\rangle = (2m\mu/\hbar^2\pi^2)^{1/2}$ is the density of the uniform non-interacting spinless system. For the SI Luttinger liquid parameters we have: $K_c(z) = 1/2$ and $v_c(z) = 2v_F(z)$. Taking the above into account the Hamiltonians for the SC Luttinger liquid can be written as

$$H_{\mathrm{SC}} = \sum_{\alpha=c,s}\frac{v_{\alpha,0}}{2}\int_{-\tilde z_\alpha(R)}^{\tilde z_\alpha(R)} d\tilde z_\alpha \left[K_{\alpha,0}\tilde\Pi_\alpha^2 + \frac{1}{K_{\alpha,0}}(\partial_{\tilde z_\alpha}\tilde\varphi_\alpha)^2\right], \quad (8)$$

where $d\tilde z_\alpha = dz/\tilde v_\alpha(z)$, $v_\alpha(z) = v_\alpha(0)\tilde v_\alpha(z) = v_{\alpha,0}\tilde v_\alpha(z)$, $\tilde\varphi_\alpha(\tilde z_\alpha) = \varphi_\alpha(z(\tilde z_\alpha))$ and $\tilde\Pi_\alpha(\tilde z_\alpha) = \tilde v_\alpha(z(\tilde z_\alpha))\Pi_\alpha(z(\tilde z_\alpha))$. One can easily check that the commutation relations of the new bose fields remain canonical. H_{SI} can be recovered from Eq. (8) proceeding as above. As we see, in the new coordinates the conformal symmetry is restored. We can therefore use this powerful property to calculate physical observables. Since the velocities vanish at the trap edges, no particle and spin current can flow out of the system and thus open boundary conditions (OBC) are effectively imposed on the system.

Now we turn to calculate the matrix correlator of density fluctuations which we shall show to differ qualitatively for different regimes of the Luttinger liquid. The choice of this observable is motivated by the versatility of trapped atomic systems, which give us the unique opportunity to independently address spin components and study correlators that are typically challenging to measure in solid state configurations [24, 25]. The matrix correlation function that we are interested in is given by

$$G_{\sigma\sigma'}(z,z') = \langle\delta\rho_\sigma(z)\delta\rho_{\sigma'}(z')\rangle, \quad (9)$$

where $\delta\rho_\sigma(z) = \rho_\sigma(z) - \langle\rho_\sigma(z)\rangle = \delta\rho_c(z)/2 + \sigma\delta\rho_s(z)$ is the fluctuation of the density of σ species. Here we consider only the smooth (non-oscillatory) part of the correlation function and rewrite the correlator in spin and charge (*i.e.*, total particle) densities

$$G_{\sigma\sigma'}(z,z') = \frac{1}{4}\langle\delta\rho_c(z)\delta\rho_c(z')\rangle + \sigma\sigma'\langle\delta\rho_s(z)\delta\rho_s(z')\rangle,$$

where we have used the property of spin-charge separation to drop the correlations between the spin and charge densities. Using $\delta\rho_c(z) = \sqrt{2/\pi}\partial_z\varphi_c(z)$ and $\delta\rho_s(z) = \sqrt{1/2\pi}\partial_z\varphi_s(z)$, the correlators are straightforwardly calculated. This is achieved by using the conformal transformation $w' = (L/\pi)\ln w - iL/2$ ($L = 2\tilde z_\alpha(R)$), which maps the upper half complex plane ($w = v\tau + iz$; $0 \le z$) (in the imaginary time formulation), with an OBC imposed at $z = 0$, to a strip of width L ($w' = v\tau' + iz'$; $-L/2 \le z' \le L/2$) [26]. Introducing the functions $\Delta_\alpha^\pm = \frac{\pi(\tilde z_\alpha(z)\pm\tilde z_\alpha(z'))}{4\tilde z_\alpha(R)}$ and $\tilde R_\alpha^2 = 64\tilde z_\alpha^2(R)\tilde v_\alpha(z)\tilde v_\alpha(z')$, the correlation functions for the SC Luttinger liquid are given by

$$G^{\mathrm{SC}}_{\sigma\sigma'}(z,z') = G_c(z,z') + \sigma\sigma' G_s(z,z'), \quad (10)$$

where

$$G_\alpha(z,z') = -\frac{K_{\alpha,0}}{\tilde R_\alpha^2}\left(\frac{1}{\sin^2\Delta_\alpha^-} + \frac{1}{\cos^2\Delta_\alpha^+}\right). \quad (11)$$

For the SI Luttinger liquid the average in the spin sector is zero for large relative coordinates ($|z - z'| \gg a_0$, where a_0 is the average distance between atoms), since the spin excitations are exponentially damped [21] and the correlator reads

$$G^{\mathrm{SI}}_{\sigma\sigma'}(z,z') = -\frac{1}{2\tilde R_c^2}\left(\frac{1}{\sin^2\Delta_c^-} + \frac{1}{\cos^2\Delta_c^+}\right). \quad (12)$$

Let us discuss these functions in more detail. Remarkably, the off-diagonal correlators ($\sigma \ne \sigma'$) show a qualitative difference between the SC Luttinger liquid and its spin-incoherent counterpart; namely, $G^{\mathrm{SC}}_{\sigma\bar\sigma}(z,z') > 0$, while $G^{\mathrm{SI}}_{\sigma\bar\sigma}(z,z') < 0$. (Notice that for a noninteracting system $G^{\mathrm{NI}}_{\sigma\bar\sigma}(z,z') = 0$.)

Having obtained the correlation function for a single tube we are in a position to describe the response from the array of tubes, which is more faithful to the experimental situation. Since we assume that there is no particle transfer between the tubes, we can conclude that

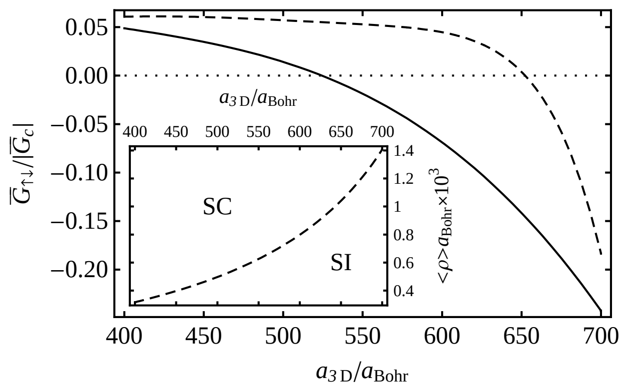

FIG. 2: Qualitative behavior of $\bar{G}_{\uparrow\downarrow}/\left|\bar{G}_c\right|$ as a function of the 3D scattering length, $a_{3\mathrm{D}}$, in a harmonic trap for ^{6}Li atoms ($N = 10^5$,$\omega_\perp = 2160\pi$Hz, $\omega_0 = 14\pi \times 10^5$Hz and $\omega_z = 470\pi$Hz) for the central tube (*dashed*) and the array of tubes (*solid*). As the interaction strength is increased, the response changes to negative, indicating the crossover from the spin-coherent Luttinger liquid regime to its spin-incoherent counterpart. *Inset*: "Phase diagram" of a 1D ultra-cold atomic Fermi gas, the dashed curve is a crossover line between the two regimes.

they are uncorrelated. Thus, the many-tubes correlation function can simply be written as

$$G^{ij}_{\sigma\bar{\sigma}}(z,z') = \left\langle \delta\rho^i_\sigma(z)\delta\rho^j_{\bar{\sigma}}(z')\right\rangle = \delta_{ij}G^{ii}_{\sigma\bar{\sigma}}(z,z'), \qquad (13)$$

where the superscripts i and j label different tubes. The responses from different tubes simply add up. In the *in situ* experiment, when the measuring laser beam probes the whole array of tubes, one is able to measure the full integrated response, which is given by $\bar{G}_{\sigma\bar{\sigma}} = \sum_i \int_{-R}^{R} G^{ii}_{\sigma\bar{\sigma}}(z,z')dzdz'$; this averaging provides an observable with a good signal-to-noise ratio. In Fig. 2, we show the qualitative behavior of $\bar{G}_{\uparrow\downarrow}/\left|\bar{G}_c\right|$ for an array of tubes confined in a global harmonic trap. We have used realistic experimental parameters for ^{6}Li atoms to see that it is possible to identify the response for the two regimes without pushing the values to extreme limits. We see that the SI Luttinger liquid regime is realized (as expected) for strong interactions and thus yields the negative response, while the positive signal for weak interactions is due to the SC Luttinger liquid correlations.

In conclusion, we have proposed an experiment to measure signatures of strong interactions in 1D ultra-cold atomic systems. We have shown that correlations of density fluctuations (an observable easily accessible in atomic physics experiments compared to their condensed matter counterparts) qualitatively distinguish between different strongly correlated regimes in these systems and therefore provide an ideal probe for detecting these regimes in cold-atom experiments. While the off-diagonal correlators are positive for the spin-coherent Luttinger liquid, they are negative for its spin-incoherent counterpart. This result is robust, does not depend on the details of the trap and should be easy to identify in the experiment. This is also a novel proposal to measure properties of the spin-incoherent regime in a non-condensed matter system, which opens up new avenues for the study of Luttinger liquids. In particular, it would be interesting to extend these studies to the spin-imbalanced case.

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